# Creep Behavior of Salt in Triaxial Extension Tests

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## INTRODUCTION

It has been established from field observations and laboratory work that the time dependent response of salt is a most significant aspect of its behavior under load. This paper presents and discusses the results obtained from creep tests on salt. In order to place the subsequent discussion in proper prospective, it is necessary to review briefly some of the concepts involved in the selection of a mathematical representation for the response of a material to load. Such a representation is termed a constitutive equation or a mathematical model.

## THEORETICAL CONCEPTS

The purpose of selecting a mathematical model to represent material response is to be able to formulate and solve a boundary value problem from which the response variables (e.g., stress, strain, deflection), can be calculated if the input variables (e.g., loads, temperature) are defined. The model should reflect the mechanical, thermal, and hygroscopic properties of the material, and should include construction procedures and possible changes with time and input. Modeling at this level of generality is not possible. Some variables have to be kept fixed or disregarded in setting up the model; this involves selection of those variables which are of importance to a particular problem. If it is assumed that the input variables have been specified, it then becomes necessary to introduce various physical postulates concerning the response of the material. These are concerned with the formulation of a constitutive equation for a material.

The following postulates are introduced:

1. The mechanical state is defined by the stress and strain matrices and the thermal and moisture states are defined by temperature and moisture content, respectively.
2. The stress in the material at time "t" is a function of the histories of strain, temperature, and moisture content at that point and a function of the age of the system.

Postulate (2) is the basis for formulating a fundamental constitutive equation to determine the response of the material to a variety of inputs. All mathematical models which are currently in use with theoretical methods of analyzing problems in rock mechanics appear as special cases of this postulate. In addition, however, many other less restrictive types of models are also included. It will be convenient to introduce some notations at this point so that the constitutive equation can be represented in mathematical form:

| | |
|---|---|
| $\bar{x}$ | Position vector of a point, referred to a coordinate system |
| $\bar{\sigma}(\bar{x}, t)$ | Stress, a matrix function of space and time |
| $\bar{\epsilon}(\bar{x}, t)$ | Strain, a matrix function of space and time |
| $\bar{T}(\bar{x}, t)$ | Temperature change above a reference state, a scalar function of space and time |

$\bar{M}(\bar{x}, t)$ Moisture content, change above a reference state, a scalar function of space and time

Postulate (3) is equivalent to the statement:

$$\bar{\sigma}(\bar{x}, t) = \bar{\bar{F}}_{s=0}^{s=t}\left[\bar{\epsilon}(\bar{x}, s), \bar{T}(\bar{x}, s), \bar{M}(\bar{x}, s); \bar{x}, t\right] \tag{1}$$

The notation infers that stress is a function of the histories of the variables shown from time zero to current time "t". Since stress is a function of arguments, which in turn are functions, it is customary to refer to this equation as a functional, i.e., stress is a functional of strain, temperature, and moisture content. In general, determination of the temperature and moisture content as functions of time cannot be carried out independently of the determination of mechanical variables, i.e., the variables are *coupled*. Due to the complexity of such theories, it is generally assumed that the temperature and moisture content variables are known functions of space and time. These functions can be found from the solution of separate diffusion boundary value problems. Equation 1 can be specialized further if the material is supposed to have symmetry in its behavior, i.e, whether the material is anisotropic or whether it is isotropic. It should be noted that the function F in Equation 1 depends on position "x" and time "t". The dependence of the properties on position admits the possibility of a non-homogeneous system. When F depends on "t", the material has aging characteristics, i.e., the properties depend on age. If F is independent of age, or *time invariant*, the time argument disappears and only relative time "t-s" appears in the histories. Thus, for a nonaging homogeneous material we have:

$$\bar{\sigma}(\bar{x}, t) = \bar{F}_{t-s=0}^{t-s=t}\left[\bar{\epsilon}(\bar{x}, t-s), \bar{T}(\bar{x}, t-s), M(\bar{x}, t-s)\right] \tag{2}$$

This equation includes all models currently in use for rock mechanics problems, e.g., elasticity and viscoelasticity.

The constitutive equation (2) is said to be hereditary in the sense that the current value of stress depends upon the *history* of the argument functions. It is possible to idealize material response by assuming it is the same for all histories, i.e., the stress depends only upon the current values of the arguments. In other terms, the response of the system is path independent for all of its input variables. This implies that all effects are completely reversible; there cannot be any hysterisis or permanent deformation, such a material is called *elastic*. It must be recognized that *elastic theory* cannot, by *definition*, predict time-dependent, rate-dependent or permanent deformation.

Some materials, when loaded beyond a critical state of stress called the "yield condition," undergo a permanent deformation so that upon unloading they do not return to their initial state. It is customary to assume that this post-yielding deformation *is not dependent upon time*, rather it depends upon the stress path. Such a constitutive law cannot predict time dependent effects.

It was noted that in a hereditary material, the present values of stress depends upon the history of each of the argument variables. If the response of mechanical, thermal, or moisture inputs is symmetric in some sense, the material is said to have that class of symmetry relative to a particular input variable. Complete symmetry is called isotropy. However, at this point the possibility of applying Equation 2 to experiments from which the unknown response functions can be derived is quite remote.

In order to model time dependent behavior one is primarily concerned with a viscoelastic model. To draw attention to the character of viscoelastic response, without overly limiting the applicability of the discussion, it will be assumed that in Equation 2 the stress depends on the history of strain and upon the present values of temperature and moisture content, i.e., stress is a functional of strain and a function of T, M. With this restriction, Equation 2 takes the form:

$$\bar{\sigma}(\bar{x}, t) = \bar{F}^{t}_{s=0}\left[\bar{\epsilon}(\bar{x}, t-s), \bar{T}(\bar{x}, t), \bar{M}(\bar{x}, t)\right] \tag{3}$$

It should be noted that linearity has not been introduced. To represent a material in accordance with Equation 3, a sequence of time, temperature, and moisture dependent kernel functions which must be determined experimentally are utilized. At the present time (1969) experimental techniques for characterizing nonlinear viscoelastic materials are not well developed. Furthermore, methods of solving boundary value problems of the generality required for the evaluation of the majority of rock mechanics problems using a nonlinear viscoelastic model are not available, though they are under development.

In the special case that the mechanical response of a material is linear, the form of Equation 3 is greatly simplified and under constant temperature and moisture condition will reduce to the familiar Boltzman superposition integral used for representing linear homogeneous isotropic viscoelastic materials. A viscoelastic material, whether linear or nonlinear, exhibits instantaneous elastic response as well as other phenomena not found in elastic materials: stress dependence upon rate or path of straining, elastic recovery after removal of load, and the possibility of permanent deformation upon removal of load. A viscoelastic material does not exhibit a "yield" effect, there is no time-independent permanent deformation.

It has been observed in salt that the time-dependent response is significantly nonlinear, and cannot be satisfactorily represented by linear constitutive laws. Furthermore, it has not been shown that the time dependent deformation that does occur is viscous or that all instantaneous deformation is elastic; necessary conditions for the applicability of viscoelasticity. In view of the present state of the art in nonlinear viscoelastic experimentation and stress analysis, nonlinear time dependent response is best characterized by empirical relations. The use of empirical relations to characterize time-dependent behavior is widely used in structural analysis where metals are subjected to loads at elevated temperatures resulting in significant time dependent deformation (Finnie and Heller, 1959; Odqrrst, 1966; Manson, 1968). It is this approach that is followed in this paper in modeling the time dependent response of salt. When empirical relations are utilized for stress analysis the time independent and time dependent effects are treated independently and their effects superimposed. This is not viscoelasticity, however, where linear viscoelasticity cannot represent material behavior, analysis based on the superposition of time independent and time dependent effects have been used with considerable success in structural analysis.

The results of a typical creep test and the terminology commonly used to describe the strain-time curve are shown in Figure 1. In many cases, the secondary portion of the creep curve can be considered to represent the major portion of any creep deformation. In such cases, the strain can be considered to consist of an initial elastic strain and a creep strain which increases at a constant rate. The creep strain can, therefore, be represented by straight lines whose slopes are in general stress dependent, indicating the nonlinear character of the material. A commonly used relationship between creep strain and stress under these conditions is of the form*:

$$e_c \text{ (creep strain)} = B\sigma^n t \tag{4}$$

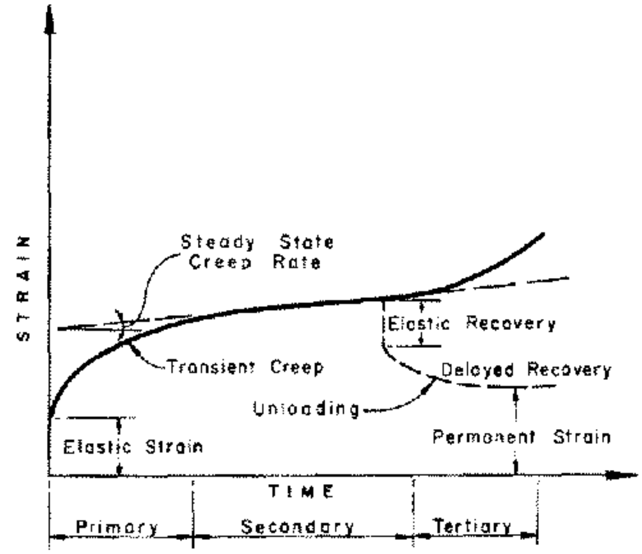


**Figure 1.** Schematic representation of results from a creep test.

$$\dot{e} \text{ (creep strain rate)} = B\sigma^n \tag{5}$$

It is generally assumed that B is a function only of temperature; i.e., independent of stress and strain, and n is a constant. Other relationships between strain, stress, and time that have been utilized include:

$$e_c = Ae^{\sigma/B}t \tag{6}$$

and

$$e_c = A\sigma^n t + B\sigma^m t \tag{7}$$

If the transient portion of the creep curve is large, then the idealizations proposed above are not appropriate. However, it has been observed that the creep curves for various stress levels have a geometrical similarity which allows the creep strain to be expressed in the form:

$$e_c = H(\sigma)\,F(t) \tag{8}$$

An expression of this type commonly used (Manson, 1968; Finnie, 1966) in stress analysis for creep is:

$$e_c = A\sigma^n t^m \tag{9}$$

where A, n, and m are constants.

*The dot above e denotes differentiation with respect to time, i.e., $\dot{e}_c = \frac{de_c}{dt}$.

Obviously, creep strain-stress-time relations of varying degrees of complexity can be set up; however, because of the sensitivity of the creep behavior to temperature, stress conditions, choice of sample, and testing techniques, a simple empirical relation (e.g., Equation 9) may be the best choice (Finnie, 1966).

Test results, in most cases, relate one component to stress to one component of strain. For purposes of stress analysis it is necessary to obtain a relation between general stress and strain states. In order to make this generalization, it is often assumed that the relationship between effective stress and effective strain obtained in a laboratory test under special stress conditions is valid under all stress states (Manson, 1968; Finnie, 1966). For a uniaxial test, the effective stress and strain are the axial stress and strain respectively. For a triaxial extension test, the stress difference $(\sigma_3 - \sigma_1)$ and the axial strain are the effective stress and strain respectively. The incorporation of the stress-strain-time law into the solution of a boundary value problem can be done in many ways (Manson, 1968; Finnie, 1966; Boresi and Deere, 1963; Nair, 1967). Such a discussion is outside the scope of this paper. Equation 9 was utilized by Boresi and Deere (1963) and Nair (1967) in representing the creep data available from triaxial extension tests on salt and in analyzing various boundary value problems in rock mechanics. The information obtained from the creep tests conducted in various investigations will be discussed in the context of the relationship given in Equation 9.

## DISCUSSION OF TEST RESULTS

Triaxial extension creep tests were conducted on the salt by the Corps of Engineers (1963) and reported by Boresi and Deere (1963). Additional results have been conducted and reported by Nair (1968).

If Equation 9 is used as a basis for discussion, then it is necessary to determine the constants and establish if the data can be satisfactorily represented by Equation 6. In order to determine the various constants, Equation (9) is recast into logarithmic form as follows:

$$\log \epsilon_c = \log A + n \log \sigma + m \log t \tag{10}$$

It can be seen that for a fixed value of , the above equation can be written in the form:

$$\log \epsilon_c = K + m \log t \tag{11}$$

Therefore, a plot of strain versus time on a logarithmic scale would result in a straight line from which m could be determined.

If time is kept fixed, Equation (10) can be rewritten as:

$$\log \epsilon_c = K_1 + n \log \sigma \tag{12}$$

A plot of $\epsilon_c$ against $\sigma$ on a logarithmic scale should also result in a straight line from which n can be determined. Knowing n and m, A can be readily determined from Equation (9).

Figure 2 represents the data obtained in the triaxial extension creep tests. It can be observed that the data plot fairly consistently as straight lines and that the slopes of the various lines can be grouped in two categories giving values of m = .318 and .446. An average value of .382 can be considered satisfactory for analysis and does not differ significantly from the value of .36 used in earlier studies (Boresi, 1963). This indicates that Equation 11 is a satisfactory representation of the data.

In order to determine the constant n, strain and stress were plotted at a constant time as shown in Figures 3 through 6. The results can be satisfactorily represented by straight lines, which indicates that Equation 12 is a valid method of representing the data.

It should be recognized that the computation of n depends on how the data is fitted with a straight line. Since a great deal of the data is concentrated in a narrow band of stress difference and because of the steepness of the slopes of these lines, it is certainly more appropriate to talk of ranges in the values of n rather than any specific value. An attempt was made to determine if the first stress invariant had a significant influence on the magnitude of the constants in Equation 4. Typical results isolating the effect of the first stress invariant is shown in Figure 7. While there is the possibility of the first stress invariant influencing the behavior, it is doubtful if the data available at the present time can be considered sufficient to derive any conclusions. However, there appears to be sufficient data to warrant further investigation of the problem.

The constant A is determined from Equation (9), utilizing the selected values of n and m. Since n is much greater than 1.0, the numerical value of n greatly influences the magnitude of A. Therefore, if n is influenced by the first stress invariant, A will also be subject to this influence. Values of all the constants are presented in Figures 3 through 6.

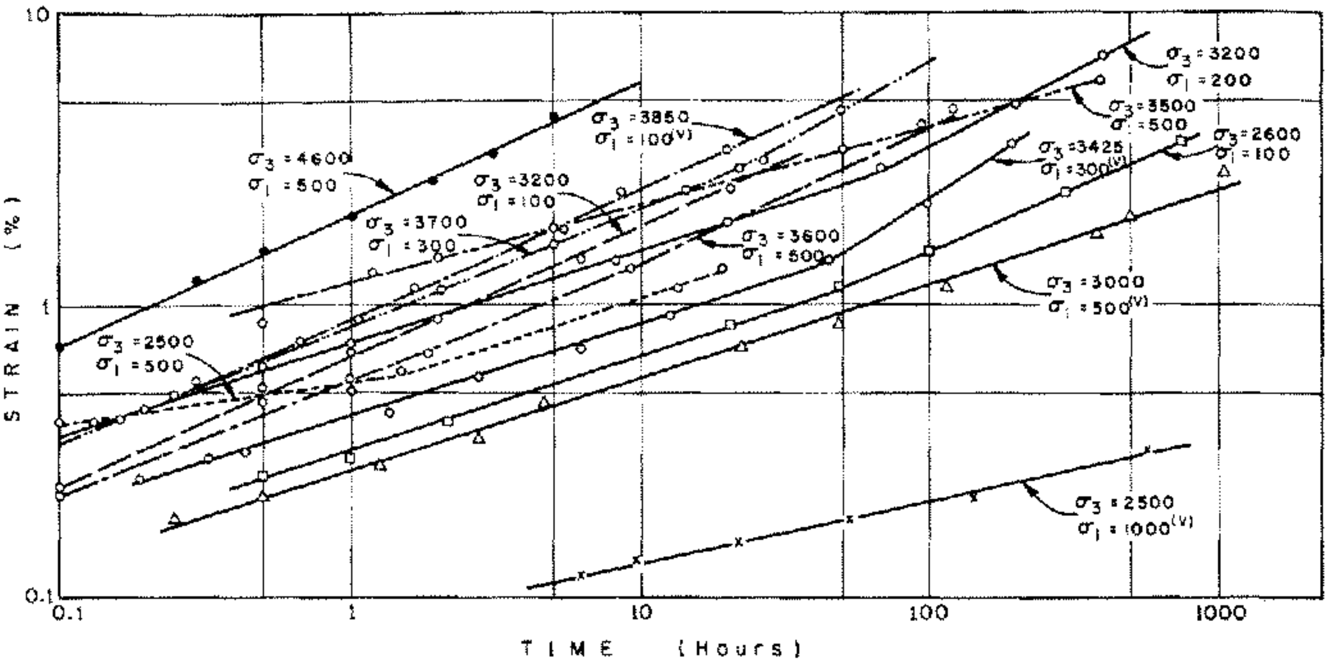


Figure 2. Creep strain–time curves–logarithmic plot.

## FINAL REMARKS

General concepts of choosing constitutive equations for representing material response have been discussed. It has been pointed out that for materials like salt which exhibit a high degree of nonlinear time dependent effects, constitutive equations which satisfy the rigorous requirements of solid mechanics and for which meaningful stress analysis can be performed are presently not available. It is, therefore, necessary to use empirical relations to represent material response, such empirical relations have been used successfully in structural analysis.

The results of the creep tests plotted in Figures 5 through 8 do indicate that a creep equation of the form $\epsilon_c = A\,\sigma^n t^m$ is *one* equation that can be used to represent the time dependent response of salt. Such an equation, when combined with elastic stress-strain laws, can be used to solve boundary value problems of interest in rock mechanics.

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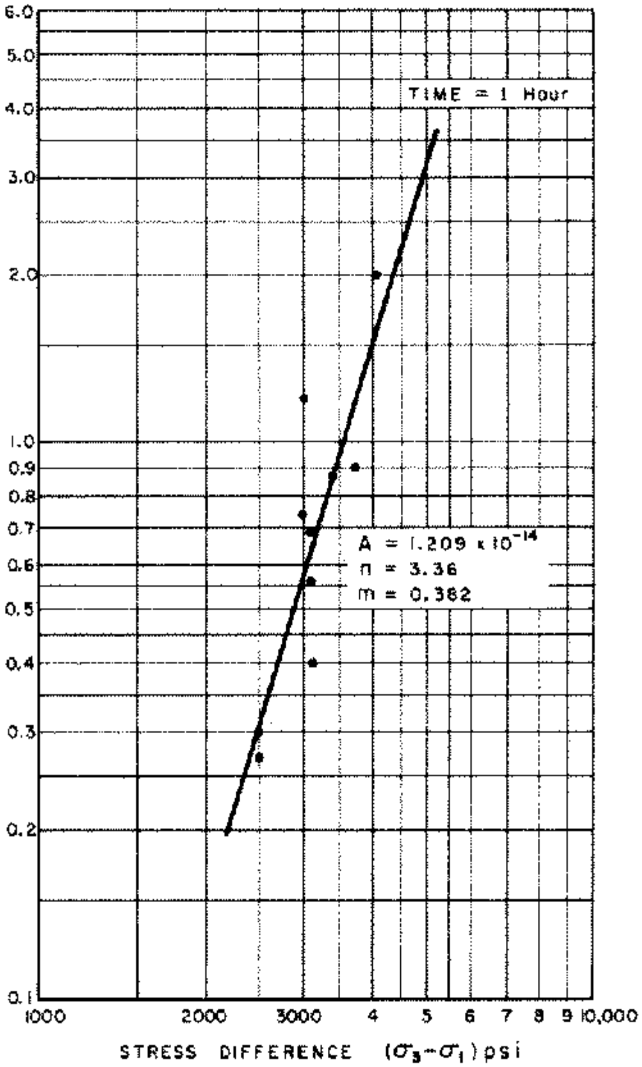


gure 3. Strain as a function of stress difference for a constant ne.

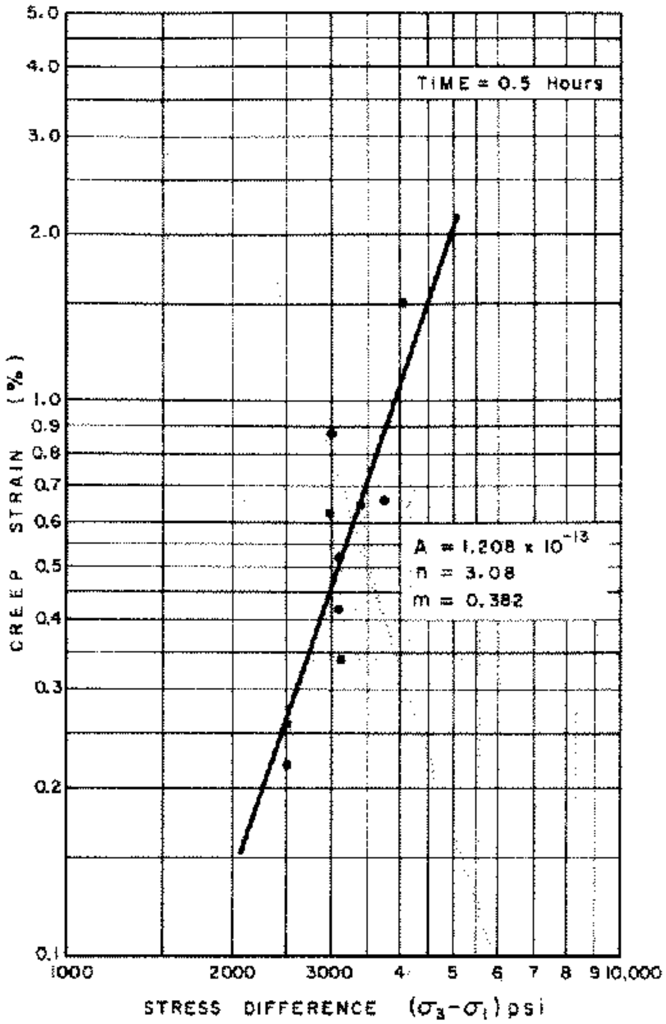


Figure 4. Strain as a function of stress difference for a constant time.

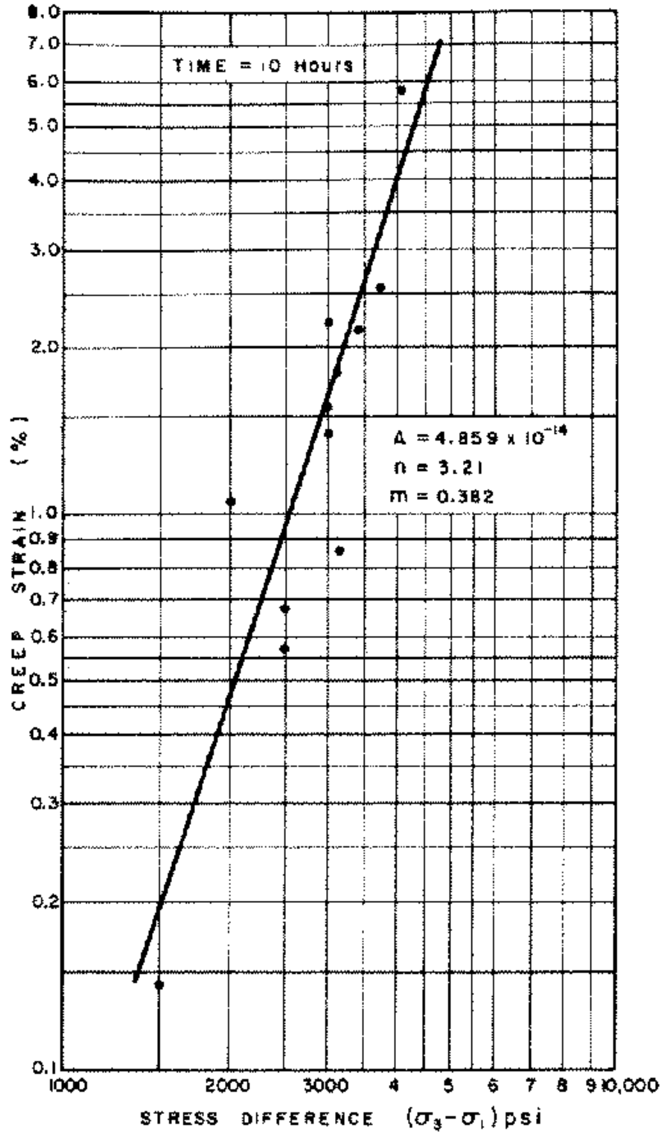


**Figure 5.** Strain as a function of stress difference for a constant time.

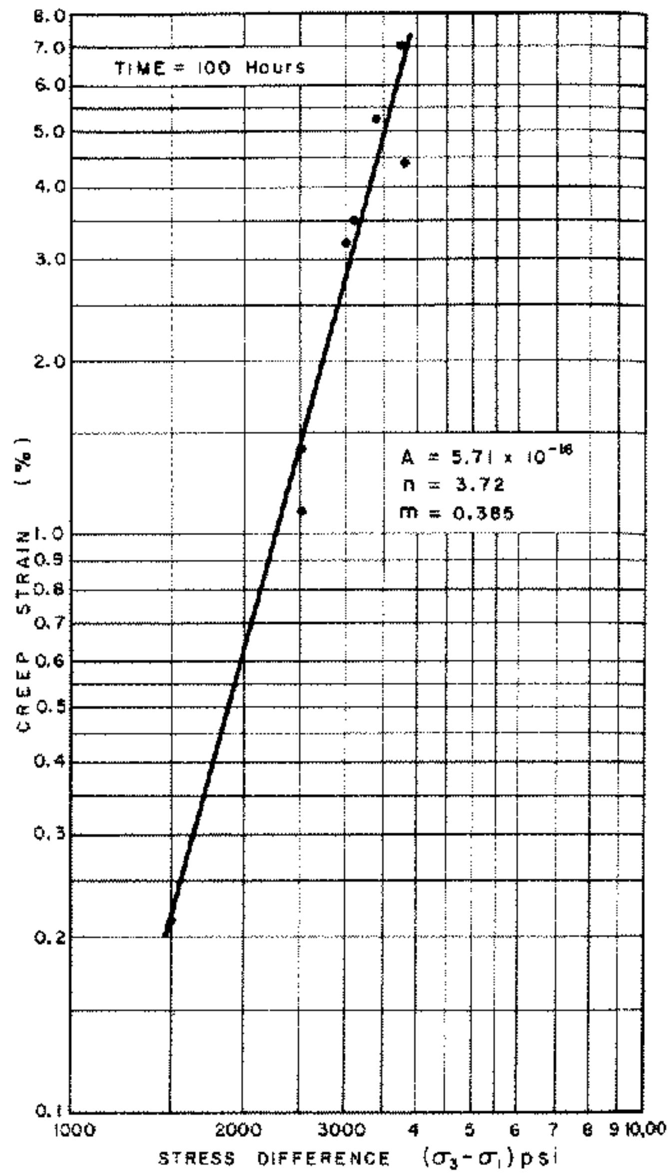


**Figure 6.** Strain as a function of stress difference for a constant time.

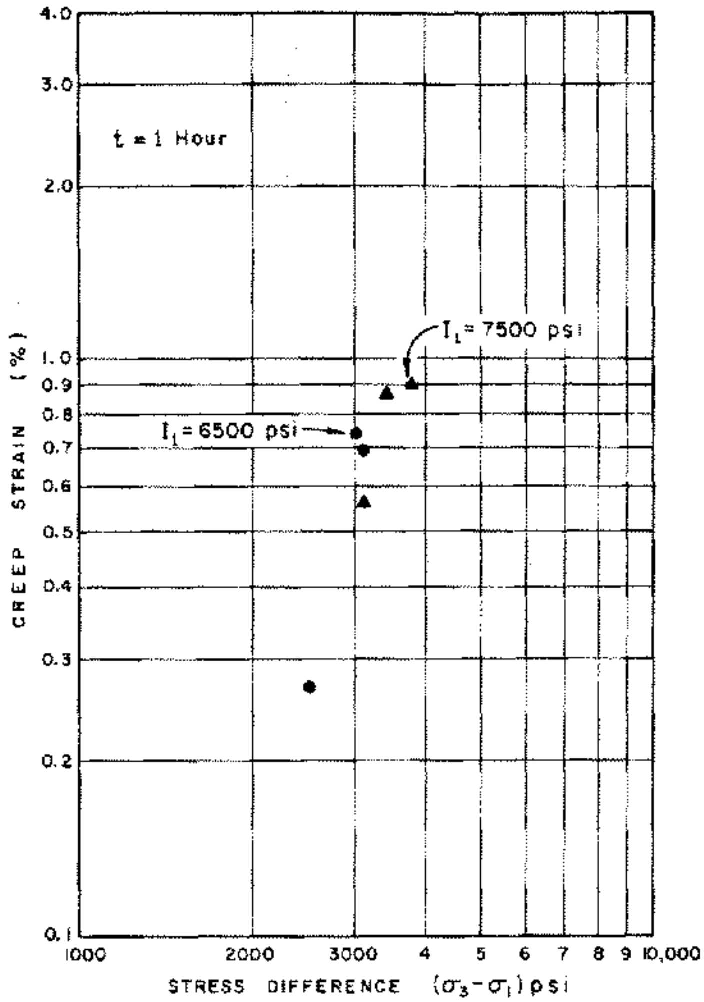


**Figure 7.** Strain as a function of stress difference for a constant time. Influence of first stress invariant.